5.1.4 Natural light

The electromagnetic waves emitted from any common source of light are **polarized randomly** or **unpolarized**. That is, the electric field changes directions randomly.

We can use the mess like that in Fig. 5.6 (a) to represent the unpolarized light. In principle, we can simplify the mess by resolving each electric field in Fig. 5.6 (a) into y and z components and then finding the net fields along the two directions. In doing

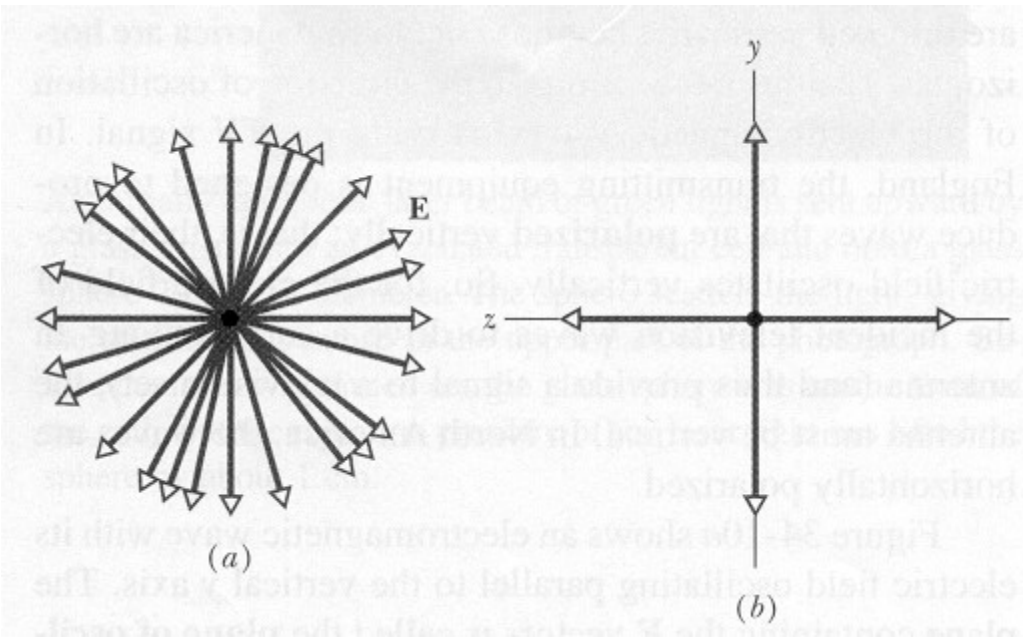

Fig. 5.6 (a) and (b) Two different drawings to represent natural light.

so, we mathematically change unpolarized light into the superposition of two polarized waves whose planes of oscillation are perpendicular to each other. The result is the double-arrow representation of Fig. 5.6 (b), which simplifies drawing of unpolarized light. Actually, light is generally neither completely polarized nor completely unpolarized. More often, the electric field vector varies in a way that is neither totally regular nor totally irregular, and one refers to such an optical disturbance as being partially polarized. For this situation, we can draw one of the arrows of the double-arrow representation longer than the other arrow.

5.2 Polarizers

An optical device whose input is natural light and whose output is some form of polarized light is a polarizer.

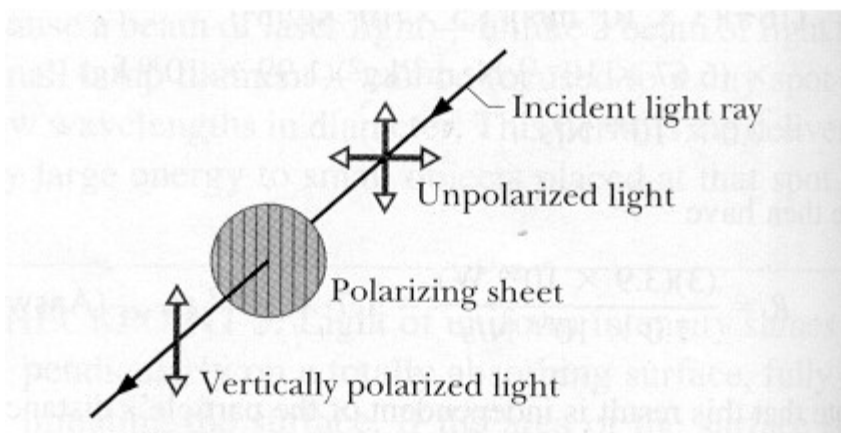

Fig. 5.7 A linear polarizer.

Fig. 5.7 shows a linear polarizer. Depending on the form of the output, we could also have circular or elliptical polarizers.

Polarizing direction of a linear polarizer: <u>An electric filed component parallel to the polarizing direction is passed (transmitted) by a polarizer; a component perpendicular to it is absorbed.</u>

Intensity of transmitted light through a linear polarizer:

When unpolarized light reaches a polarizer, the intensity of the emerging polarized light is then

$$I = \tfrac{1}{2} I_0 . \qquad (5.19)$$

Now we suppose that the light reaching a polarizer is already polarized. Fig. 5.8 shows a linear polarizer and the electric-field $\vec{E}$ of such a polarized light traveling toward the polarizer. We can resolve $\vec{E}$ into two components relative to the polarizing direction of the polarizer: parallel component E_y is transmitted by the polarizer, and the perpendicular component E_z is absorbed. Since θ is the angle between $\vec{E}$ and the polarizing direction, the transmitted parallel component is

$$E_y = E\cos\theta . \qquad (5.20)$$

Recall that the intensity (irradiance) of an electromagnetic wave is proportional to the square of the electric-field's magnitude. In our present case then, the intensity I of the emerging wave is proportional to E_y^2 and the intensity I_0 of the original wave is proportional to E^2. Hence, from Eq. 5.20 we can write

$$I = I_0 \cos^2 \theta. \qquad (5.21)$$

This is knows as Malus's law. We also call this the cosine-sequared rule.

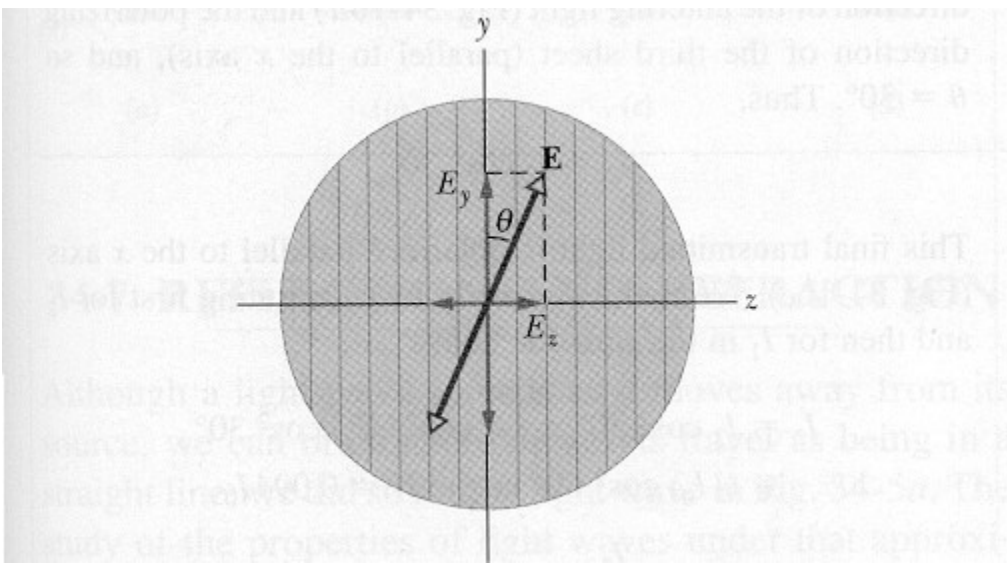

Fig. 5.8 Polarized light approaching a linear polarizer.

Fig. 5.9 shows an arrangement in which initially unpolarized light is sent through two polarizers P_1 and P_2. (often, the first polarizer is called the polarizer, and the second the analyzer.) If their polarizing directions are parallel, all the light passed by the first polarizer is passed by the second polarizer. If those directions are perpendicular (the polarizers are said to be crossed), no light is passed by the second polarizer.

Finally, if the two polarizing directions make an angle between 0° and 90°, some of the light transmitted by P_1 will be transmitted by P_2. The intensity of that light is determined by Eq. 5.21.

We can use the setup of Fig. 5.9 along with Malus's law to determine whether a particular device is linear polarizer.

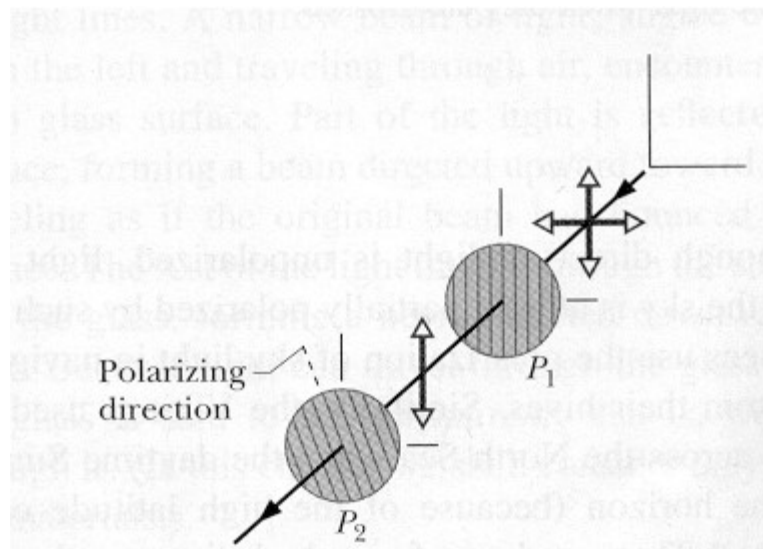

Fig. 5.9 A linear polarizer and analyzer.

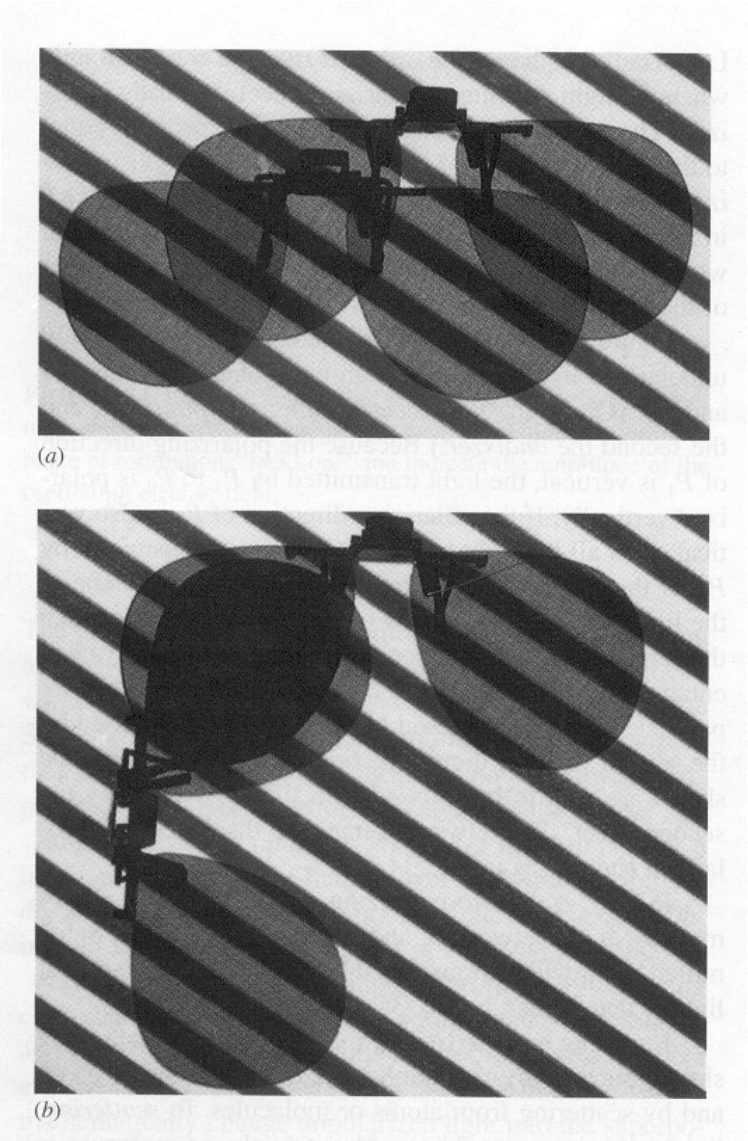

Fig. 5.10 Polarizing sunglasses consist of sheets whose polarizing directions are vertical when the sunglasses are worn. (a) Overlapping sunglasses transmit light fairly well when their polarizing directions have the same orientation, but (b) they block most of the light when ther are crossed.

5.3 Birefringence

5.3.1 birefringence

The refractive index n is described in Eq. (3.44), where ω and ω_0 are the angular frequency of the driving force and the natural angular frequency of the electric dipole, respectively. In a isotropic material, ω_0 is a constant along all directions. A material can have an anisotropy in the bound force as shown in Fig 5.11. It leads to different ω_0 and therefore different refractive index n along different directions. If the stiffness of the spring in x direction has one value while has another value along any direction perpendicular to x-direction, the material has two different refractive indices and is said to be **birefringent**. The x-direction is called the **optic axis**. Any plane contains the optic axis is called the **principal section**. The birefringent property is determined by the material's atomic structure.

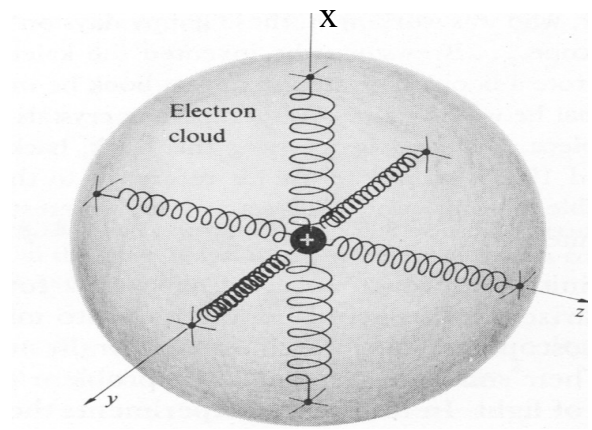

Fig 5.11 mechanical model depicting a negative charged shell bound to a positive nucleus by pairs of springs having different stiffness

Fig 5.12 shows what we can see through a calcite crystal ($CaCO_3$): two images of an object. One of the image is formed by the **ordinary rays (o-rays)** with linear polarization perpendicular to the principal section (fig 5.13). Another image is formed by the **extraordinary rays (e-rays)** with linear polarization parallel to the principal section (Fig 5.14).

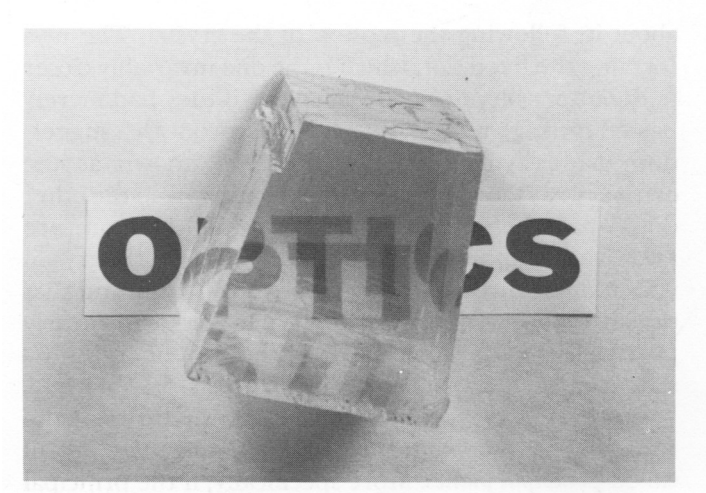

Fig 5.12 Double image formed by a calcite crystal

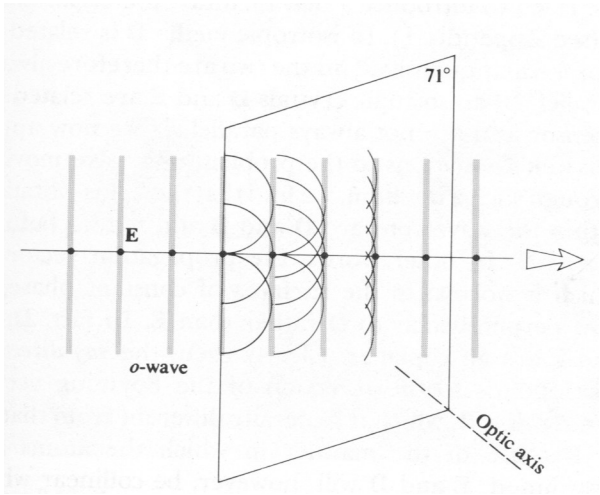

Fig 5.13 An incident plane wave polarized perpendicular to the principal section (o-ray)

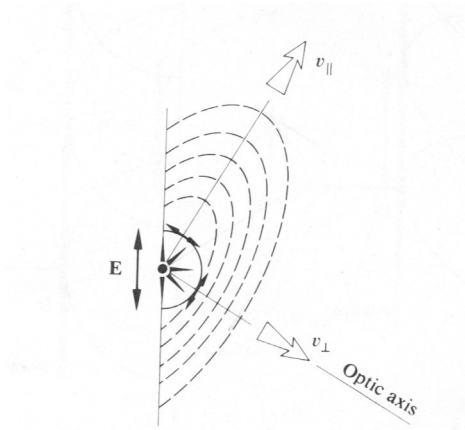

Fig 5.15 wavelets within calcite for e-rays

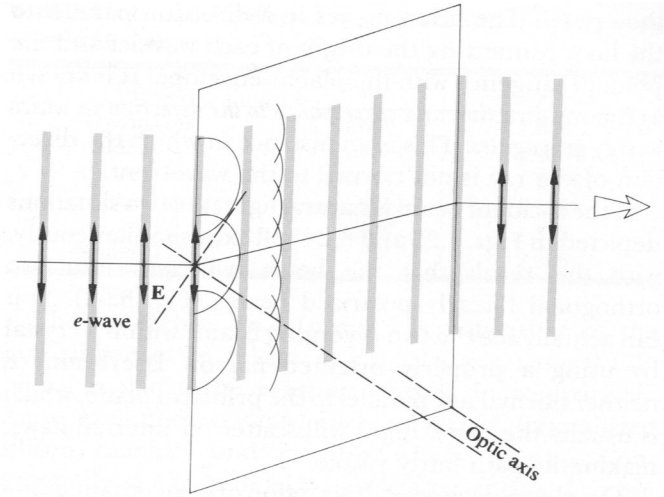

Fig 5.14 An incident plane wave polarized parallel to the principal section (e-ray)

In the case of o-ray (Fig 5.13), the **E**-field and the wavelets is everywhere normal to the optic axis, they will expand into the crystal in all directions with a speed $v_{\perp}$as they would in an isotropic medium. In contrast, in the case of e-ray (Fig. 5.14), the **E**-field has a component normal to the optic axis, as well as a component parallel to it. Since the medium is birefringent, light polarized parallel to the optic axis propagates with a speed $v_{\parallel}$, where $v_{\parallel} > v_{\perp}$ in the case of calcite. Therefore the wavelet will elongate in all directions normal to the optic axis, forming wave fronts of ellipsoids of revolution about the optic axis as shown in Fig 5.15. The envelope of all the ellipsoidal wavelets is still a plane wave parallel to the incident wave (Fig 5.14). However, the beam moves in a direction parallel to the lines connecting the origin of each wavelet and the point of tangency with the planar envelope. Therefore the e-ray travels a different way in the crystal, forming double image.

Crystals belonging to the hexagonal, tetragonal and trigonal systems have their atoms arranged so that light propagating in some direction will encounter an asymmetric structure. Such substances are optically anisotropic and birefringent. The optic axis corresponds to a direction about which the atoms are arranged symmetrically. Crystals like these, for which there is only one such direction, are known as uniaxial. o-ray is everywhere normal to the optic axis, so it moves at a speed $v_{\perp}$ in all directions, corresponding to $n_o \equiv c/v_{\perp}$. e-ray has a speed $v_{\perp}$ only in the direction of the optic axis and has a speed $v_{\parallel}$ normal to the optic axis, corresponding a refractive index $n_e = c/v_{\parallel}$. Table 5.1 shows some uniaxial crystals. Crystals with $n_e - n_o < 0$ $(v_{\parallel} > v_{\perp})$

Table 5.1 Refractive indices of some uniaxial birefringent crystals (λ_0 =589.3 nm)

Crystal	n_o	n_e
Tourmaline	1.669	1.638
Calcite	1.6584	1.4864
Quartz	1.5443	1.5534
Sodium nitrate	1.5854	1.3369
Ice	1.309	1.313
Rutile (TiO_2)	2.616	2.903

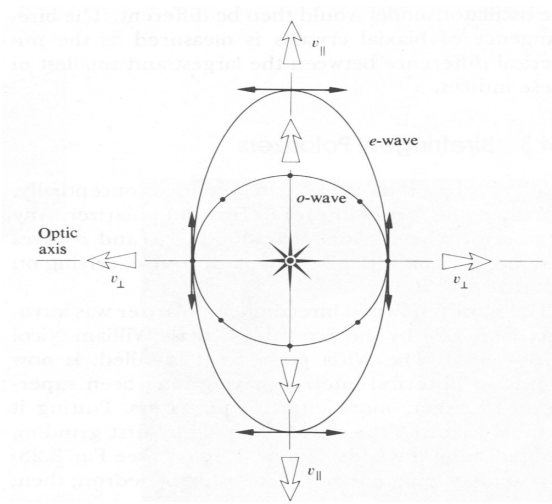

Fig. 5.16 Wavelets in a negative uniaxial crysal

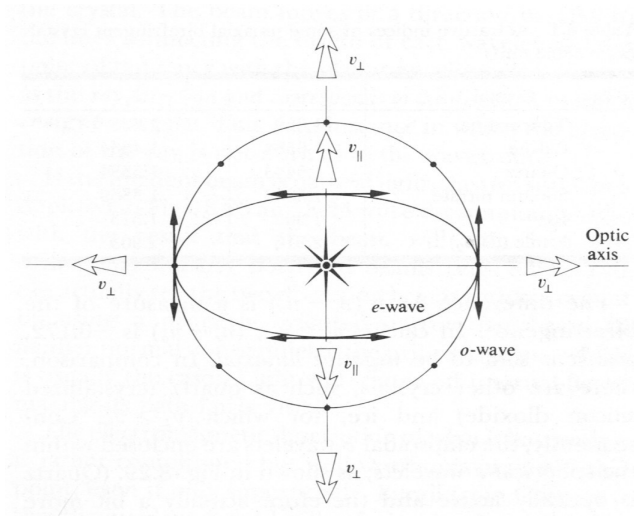

Fig. 5.17 Wavelets in a positive uniaxial crysal

are negative uniaxials, such as Calcite. The ellipsoidal e-ray wavelets are out the circular o-ray wavelets as shown in Fig. 5.16. Crystals with $n_e - n_o > 0$ $(v_{\parallel} < v_{\perp})$ are positive uniaxials, such as quarts. The ellipsoidal e-ray wavelets are enclosed within the circular o-ray wavelets as shown in Fig 5.17.

5.3.2 Birefringent polarizers

The Glan-Foucault polarizer is composed of two pieces of calcite triangles with a thin air gap in between as shown in Fig 5.18. The optic axis is perpendicular to the paper plane as being noted with dots. The incoming ray strikes the surface normally, and **E** can be resold into components that are either completely parallel or perpendicular to the optic axis. The two rays traverse the first calcite section without any deviation . Arrange the incident angle, ϑ , on the

middle calcite-air boundary to be $37.1^o = \sin^{-1}(1/n_o) < \theta < \sin^{-1}(1/n_e) = 42..3^o$. The o-ray will be internally reflected, leaving the e-ray out of the crystal. Therefore the polarization direction of the light is parallel to the optic axis.

The Wollaston prism (Fig 5.19) is a polarizing beam-splitter. It is composed of two pieces of calcite or quartz. crystals with their optic axes perpendicular to each other. The o-ray and

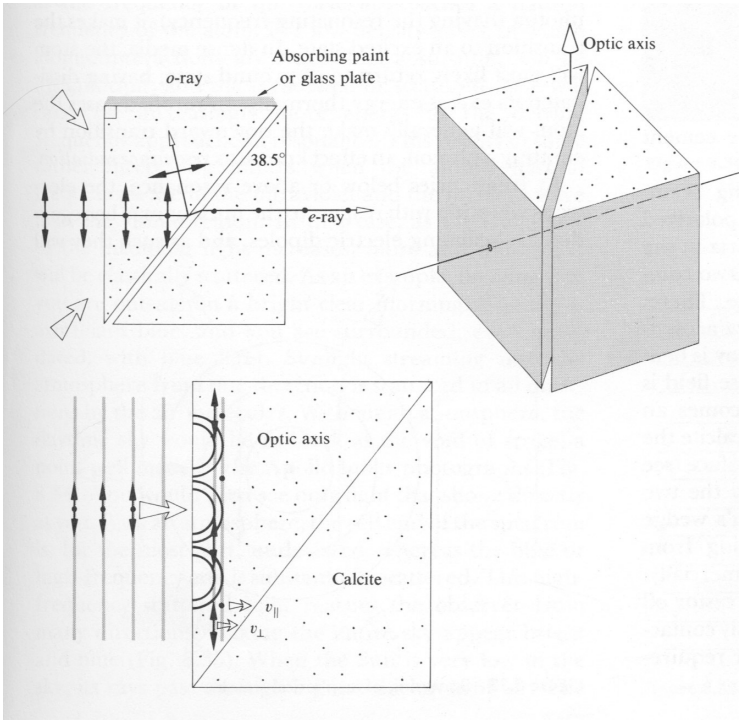

Fig 5.18 The Glan-Foucault prism

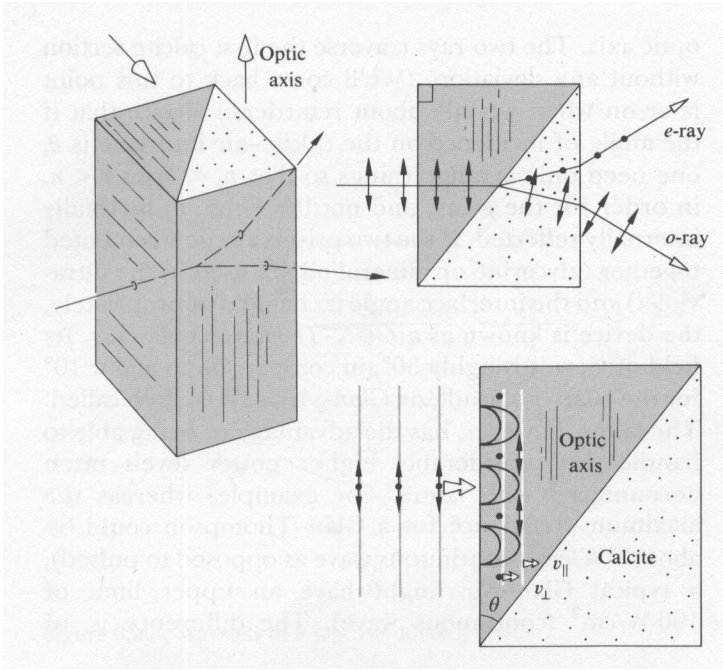

Fig 5.19 The Wollaston prism

e-ray interchange after the diagonal interface. There, the e-ray becoms an o-ray, changing its index accordingly. In calcite $n_e<n_o$, and the emerging o-ray us bent towards the normal. Similarly, the o-ray becomes an e-ray and is bent away from the normal. Out of the prism, the o-ray and e-ray are spited.

5.3.3 Retarders

Cut the calcite such that its optic axis is parallel to front and back surfaces as shown in Fig 5.20. If the **E**-field of an incident monochromatic plane wave has components parallel and perpendicular to the optic axis, two separate plane waves will propagate through the crystal. Since $v_{\parallel} > v_{\perp}, n_e < n_o$, and the e-wave will move across the plate more rapidly than the o-wave. After traversing a plate of thickness d the resultant electromagnetic waves is the superposition of the e- and o-waves, which now have a relative phase difference of $\Delta\varphi$.

$$\Delta\varphi = \frac{2\pi}{\lambda_0} d(|n_o - n_e|) \qquad (5.22)$$

where λ_0 is the wavelength in vacuum. The polarization of the emergent light depends on the amplitudes of the incoming orthogonal field components and of course on $\Delta\varphi$.

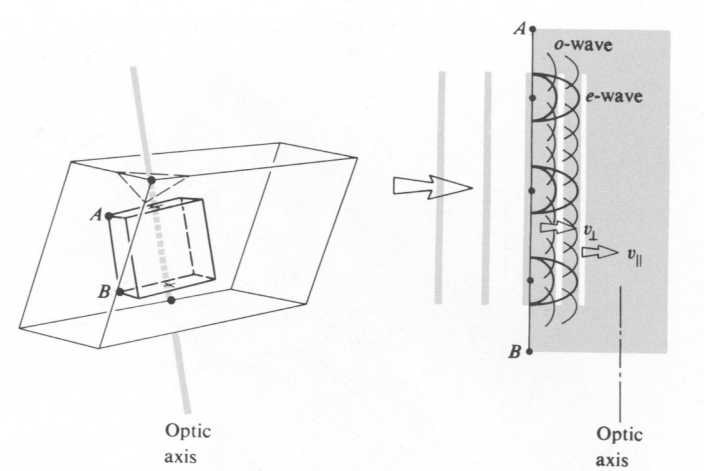

Fig 5.20 A calcite plate cut parallel to the optic axis

(A) The half-wave plate

A retardation plate that introduces a relative phase difference of 180° between the o- and e-waves is known as the half-wave plate. Suppose that the plane of vibration of an incoming beam of linear light makes an angle θ with the optical axis. As show in Fig. 5.21. In a negative material the e-wave will have a higher speed and a longer wavelength than the o-wave. When the waves emerge from the plate there will be a relative phase shift of 180°, with the effect that **E** will have rotated 2 θ. Going back to Fig. 5.5, it should be evident that a half-wave plate will similarly flip elliptical light. In addition, it will invert the handedness of circular or elliptical light, changing right to left and vice versa.

Evidently if the thickness of the material is such that $d(|n_o - n_e|) = (2m+1)\lambda_0/2$, where m is an integer, it will function as a half-wave plate.

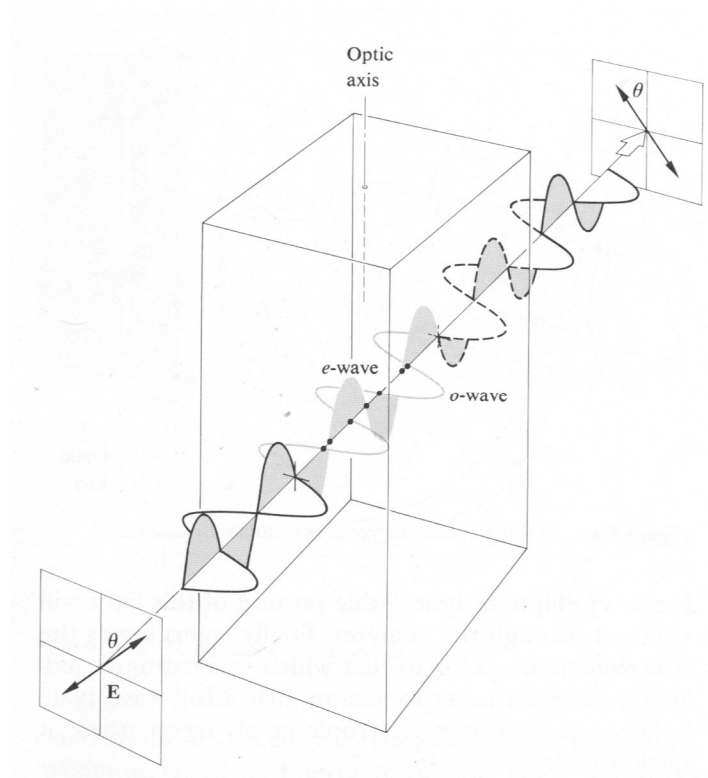

Fig. 5.21 A half-wave plate

(B) The quarter-wave plate

The quarter-wave plate is an optic element that introduces a relative phase shift of $\Delta\phi = \pi/2$ between the o- and e-wave. From Fig. 5.5, a quarter-wave plate will convert linear to elliptical light and vice versa. When linear light at 45° to the optical axis is incident on a quarter-wave plate, its o- and e-components have equal amplitudes. The out-going light will be a circular light. Similarly, an incoming circular beam will emerge linearly polarized. It should be apparent that linear light incident parallel or perpendicular to the optic axis will be unaffected by the retardation plate. You can't have a relative phase difference without having two components.

If the thickness of the material is such that $d(|n_o - n_e|) = (4m+1)\lambda_0/4$, where m is an integer, it will function as a quarter-wave plate.